Digital Authoritarianism and the Future of Human Rights

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Online Appendix

Proofs

In this section, we provide the proofs for all our propositions and results stated in the main text.

Proof of Proposition 1. The government's optimal action is the solution to the following FOC equation:

$$S'(r)G(p) - \frac{\delta C_g(r,t)}{\delta r} = 0.$$

The second order condition is $S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2} < 0$, which implies that the government's payoff is concave in its own action. The opposition group's optimal action is the solution to the following FOC equation:

$$G'(p)[1 - S(r)] - \frac{\delta C_o(p,t)}{\delta p} = 0.$$

The second order condition is $S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2} < 0$, which implies that the opposition group's payoff is concave in its own action. Because (a) each player's action set is compact; (b) each player's payoff is continuous in the other player's action; and (c) each player's payoff is concave in its own action; following Dasgupta and Maskin (1986), the game has a pure strategy Nash equilibrium. Furthermore, the equilibrium is unique because the government's reaction function $r(p)$ is strictly increasing in p, and the opposition group's reaction function $p(r)$ is strictly decreasing in r, which implies that the reaction functions can only intersect once. □

Proof of Proposition 2. Because the government's and the opposition group's best response functions are continuous in t, we can apply the implicit function theorem to see how the (interior) equilibrium actions vary with an increase in t.

The dependence of $r^*(t)$ on t is found by totally differentiating the government's and the opposition group's FOCs with respect to t, which yields the system of equations

$$S''(r)G(p)\frac{dr}{dt} + S'(r)G'(p)\frac{dp}{dt} - \frac{\delta^2 C_g(r,t)}{\delta r^2}\frac{dr}{dt} - \frac{\delta^2 C_g(r,t)}{\delta r \delta t} = 0$$

$$[1 - S(r)]G''(p)\frac{dp}{dt} - S'(r)G'(p)\frac{dr}{dt} - \frac{\delta^2 C_o(p,t)}{\delta p^2}\frac{dp}{dt} - \frac{\delta^2 C_o(p,t)}{\delta p \delta t} = 0.$$

Solving the system of equations, we get

$$\frac{dr}{dt} = \frac{\left([1 - S(r)]G''(p) - \frac{\delta^2 C_o(p,t)}{\delta p^2}\right)\frac{\delta^2 C_g(r,t)}{\delta r \delta t} - \frac{\delta^2 C_o(p,t)}{\delta p \delta t}S'(r)G'(p)}{\left([1 - S(r)]G''(p) - \frac{\delta^2 C_o(p,t)}{\delta p^2}\right)\left(S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2}\right) + [S'(r)G'(p)]^2}.$$

The denominator and numerator are both positive, and, as a result, $r^*(t)$ increases in t, as claimed.

And, solving the above system of equations for $\frac{dp}{dt}$, we get

$$\frac{dp}{dt} = \frac{\left(S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2}\right)\frac{\delta^2 C_o(p,t)}{\delta p \delta t} + \frac{\delta^2 C_g(r,t)}{\delta r \delta t}S'(r)G'(p)}{\left([1 - S(r)]G''(p) - \frac{\delta^2 C_o(p,t)}{\delta p^2}\right)\left(S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2}\right) + [S'(r)G'(p)]^2}.$$

The denominator is positive and the numerator is negative if

$$\left(S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2}\right)\frac{\delta^2 C_o(p,t)}{\delta p \delta t} + \frac{\delta^2 C_g(r,t)}{\delta r \delta t}S'(r)G'(p) < 0,$$

which implies that the $p^*(t)$ decreases in t if

$$-\frac{\delta^2 C_o(p,t)}{\delta p \delta t} < -\frac{\delta^2 C_g(r,t)}{\delta r \delta t} \cdot \frac{-S'(r)G'(p)}{S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2}},$$

and increases otherwise, as claimed. □

Proof of Proposition 3. The proof follows from the fact that S(r) increases in r and that the equilibrium level of preventive repression, $r^*(t)$ increases in t. As such, the equilibrium probability that the government stops the opposition in its infancy, $S(r^*(t))$ increases in t, as claimed. □

Proof of Proposition 4. The government's equilibrium payoff is

$$U_g^*(t) = 1 - G(p^*(t))[1 - S(r^*(t))] - C_g(r^*(t), t),$$

which implies that the effect of a change in t on the government's equilibrium payoff is given by the following expression:

$$G(p^*(t))\frac{\delta S(r^*(t))}{\delta r}\frac{\delta r^*(t)}{\delta t} - \frac{\delta G(p^*(t))}{\delta p}\frac{\delta p^*(t)}{\delta t}[1 - S(r^*(t)] - \frac{\delta C_g(r^*(t), t)}{\delta r}\frac{\delta r^*(t)}{\delta t} - \frac{\delta C_g(r^*(t), t)}{\delta t}.$$

This expression is equivalent to:

$$\frac{\delta r^*(t)}{\delta t}[G(p^*(t))\frac{\delta S(r^*(t))}{\delta r} - \frac{\delta C_g(r^*(t), t)}{\delta r}] - \frac{\delta G(p^*(t))}{\delta p}\frac{\delta p^*(t)}{\delta t}[1 - S(r^*(t)] - \frac{\delta C_g(r^*(t), t)}{\delta t}.$$

By the FOC for the equilibrium action $r^*(t)$, we have $G(p^*(t))\frac{\delta S(r^*(t))}{\delta r} - \frac{\delta C_g(r^*(t),t)}{\delta r} = 0$, which implies that the effect of a change in t on the government's equilibrium payoff is given by the following expression:

$$-\frac{\delta G(p^*(t))}{\delta p}\frac{\delta p^*(t)}{\delta t}[1 - S(r^*(t)] - \frac{\delta C_g(r^*(t), t)}{\delta t}.$$

We need to consider two cases: a) $p^*(t)$ decreases in t and b) $p^*(t)$ increases in t. In the first case, the above expression is always positive, which implies that the government's equilibrium payoff always increases in t, as claimed. In the second scenario, the government's equilibrium payoff increases in t if

$$-\frac{\delta C_g(r^*(t), t)}{\delta t} > \frac{\delta G(p^*(t))}{\delta p}\frac{\delta p^*(t)}{\delta t}[1 - S(r^*(t)],$$

where the left-hand side of the above expression represents the government's equilibrium payoff gains due to a reduction in the cost of preventive repression and the right-hand side represents the payoff losses due to an increase in the opposition's equilibrium effort.

Also, recall that $\frac{dp}{dt}$ as a function of the primitives is given by the following expression:

$$\frac{dp}{dt} = \frac{\left(S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2}\right)\frac{\delta^2 C_o(p,t)}{\delta p \delta t} + \frac{\delta^2 C_g(r,t)}{\delta r \delta t}S'(r)G'(p)}{\left([1 - S(r)]G''(p) - \frac{\delta^2 C_o(p,t)}{\delta p^2}\right)\left(S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2}\right) + [S'(r)G'(p)]^2},$$

which implies that the condition for when the government's equilibrium payoff increases in t (as a function) of the primitives (for the scenario in which $p^*(t)$ increases in t) is the following:

$$-\frac{\delta C_g(r,t)}{\delta t} > \frac{\delta G(p)}{\delta p}\frac{\left(S''(r)G(p)-\frac{\delta^2 C_g(r,t)}{\delta r^2}\right)\frac{\delta^2 C_o(p,t)}{\delta p\delta t}+\frac{\delta^2 C_g(r,t)}{\delta r\delta t}S'(r)G'(p)}{\left([1-S(r)]G''(p)-\frac{\delta^2 C_o(p,t)}{\delta p^2}\right)\left(S''(r)G(p)-\frac{\delta^2 C_g(r,t)}{\delta r^2}\right)+[S'(r)G'(p)]^2}[1-S(r)],$$

where the expression is evaluated at the (unique) equilibrium actions p^* and r^*. □

Proof of Proposition 5. In the mobilization effect game, because the government's and the opposition group's best response functions are continuous in t, we can apply the implicit function theorem to see how the (interior) equilibrium actions vary with an increase in t. The dependence of $p^*(t)$ on t is found by totally differentiating the government's and the opposition group's FOCs with respect to t, which yields the system of equations

$$S''(r)G(p)\frac{dr}{dt}+S'(r)G'(p)\frac{dp}{dt}-\frac{\delta^2 C_g(r)}{\delta r^2}\frac{dr}{dt}=0$$

$$[1-S(r)]G''(p)\frac{dp}{dt}-S'(r)G'(p)\frac{dr}{dt}-\frac{\delta^2 C_o(p,t)}{\delta p^2}\frac{dp}{dt}-\frac{\delta^2 C_o(p,t)}{\delta p\delta t}=0.$$

Solving the system of equations for $\frac{dp}{dt}$, we get

$$\frac{dp}{dt}=\frac{\left(S''(r)G(p)-\frac{\delta^2 C_g(r,t)}{\delta r^2}\right)\frac{\delta^2 C_o(p,t)}{\delta p\delta t}}{\left([1-S(r)]G''(p)-\frac{\delta^2 C_o(p,t)}{\delta p^2}\right)\left(S''(r)G(p)-\frac{\delta^2 C_g(r)}{\delta r^2}\right)+[S'(r)G'(p)]^2}.$$

Because both the numerator and the denominator are positive, this implies that $p^*(t)$ increases in t in the mobilization effect game, as claimed.

Similarly, solving the system of equations for $\frac{dr}{dt}$, we get

$$\frac{dr}{dt}=\frac{-\frac{\delta^2 C_o(p,t)}{\delta p\delta t}S'(r)G'(p)}{\left([1-S(r)]G''(p)-\frac{\delta^2 C_o(p,t)}{\delta p^2}\right)\left(S''(r)G(p)-\frac{\delta^2 C_g(r)}{\delta r^2}\right)+[S'(r)G'(p)]^2}.$$

The denominator and the numerator are positive and, as a result, $r^*(t)$ increases in t in the mobilization effect game, as claimed.

Since $r^*(t)$ is increasing in t, it follows that $S(r^*(t))$ also increases in t as claimed. □

Proof of Proposition 6. In the mobilization effect game, the government's equilibrium payoff is

$$U_g^*(t) = 1 - G(p^*(t))[1 - S(r^*(t))] - C_g(r^*(t)),$$

and the envelope theorem implies that $\frac{\delta U_g^*(t)}{\delta t} = -\frac{\delta G(p^*(t))}{\delta p}\frac{\delta p^*(t)}{\delta t}$. Since $\frac{\delta G(p^*(t))}{\delta p} > 0$ and $\frac{\delta p^*(t)}{\delta t} > 0$, this implies that $\frac{\delta U_g^*(t)}{\delta t} < 0$, as claimed. □

Proof of Proposition 7. In the preventive control effect game, because the government's and the opposition group's best response functions are continuous in t, we can apply the implicit function theorem to see how the (interior) equilibrium actions vary with an increase in t. The dependence of $r^*(t)$ and $p^*(t)$ on t is found by totally differentiating the government's and the opposition group's FOC with respect to t, which yields the system of equations

$$S''(r)G(p)\frac{dr}{dt} + S'(r)G'(p)\frac{dp}{dt} - \frac{\delta^2 C_g(r,t)}{\delta r^2}\frac{dr}{dt} - \frac{\delta^2 C_g(r,t)}{\delta r \delta t} = 0$$

$$[1 - S(r)]G''(p)\frac{dp}{dt} - S'(r)G'(p)\frac{dr}{dt} - \frac{\delta^2 C_o(p)}{\delta p^2}\frac{dp}{dt} = 0.$$

Solving the system of equations for $\frac{dp}{dt}$, we get

$$\frac{dp}{dt} = \frac{\frac{\delta^2 C_g(r,t)}{\delta r \delta t} S'(r)G'(p)}{\left([1 - S(r)]G''(p) - \frac{\delta^2 C_o(p)}{\delta p^2}\right)\left(S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2}\right) + [S'(r)G'(p)]^2}.$$

The denominator is positive and the numerator is negative, which implies that the $p^*(t)$ decreases in t as claimed.

Solving the system of equations for $\frac{dr}{dt}$, we get

$$\frac{dr}{dt} = \frac{\left([1 - S(r)]G''(p) - \frac{\delta^2 C_o(p)}{\delta p^2}\right)\frac{\delta^2 C_g(r,t)}{\delta r \delta t}}{\left([1 - S(r)]G''(p) - \frac{\delta^2 C_o(p)}{\delta p^2}\right)\left(S''(r)G(p) - \frac{\delta^2 C_g(r,t)}{\delta r^2}\right) + [S'(r)G'(p)]^2}.$$

The denominator and the numerator are positive and, as a result, $r^*(t)$ increases in t, as claimed.

Since $r^*(t)$ is increasing in t, it follows that $S(r^*(t))$ also increases in t as claimed.

□

Proof of Proposition 8. The equilibrium probability of government downfall is

$$G(p^*(t))[1 - S(r^*(t))]$$

Because the equilibrium level of preventive repression increases in t and the opposition group's equilibrium level of effort decreases in p, this implies that the equilibrium probability of government downfall always decreases in t in the preventive control effect game, as claimed.

In the preventive control effect game, the government's equilibrium payoff is

$$U_g^*(t) = 1 - G(p^*(t))[1 - S(r^*(t))] - C_g(r^*(t), t),$$

and the envelope theorem implies that $\frac{\delta U_g^*(t)}{\delta t} = -\frac{\delta G(p^*(t))}{\delta p}\frac{\delta p^*(t))}{\delta t} - \frac{\delta C_g(r^*(t),t)}{\delta t}$. Since $\frac{\delta G(p^*(t))}{\delta p} > 0$, $\frac{\delta p^*(t)}{\delta t} < 0$, and $\frac{\delta C_g(r^*(t),t)}{\delta t} < 0$, this implies that $\frac{\delta U_g^*(t)}{\delta t} > 0$, as claimed. □

Parametric Analysis. Below we prove the results of the parametric analysis section.

The Dual Effects Game. In the dual effects game, given the parametric specifications, the equilibrium actions are $p^*(t) = \frac{t}{1+t^2}$ and $r^*(t) = \frac{t^2}{1+t^2}$.[1] As such, the equilibrium probability of government downfall is given by the following expression:

$$[1 - S(r^*(t))]G(p^*(t))) = \frac{t}{(1+t^2)^2},$$

which implies that the effect of a change in t on the equilibrium probability of government downfall is given by the following expression:

$$\frac{\delta}{\delta t}\{[1 - S(r^*)]G(p^*))\} = \frac{1 - 3t^2}{(1+t^2)^3}.$$

Since $t > 0$, the equilibrium probability of government downfall increases in t if $1 - 3t^2 > 0$ and decreases in t otherwise. This implies that the equilibrium probability of government

[1] Notice that for any $t > 0$, the equilibrium actions are always interior, i.e., $0 < r^* < 1$ and $0 < p^* < 1$.

downfall increases in t if and only if

$$t \leq \bar{t} = \sqrt{\frac{1}{3}},$$

and decreases in t if $t > \bar{t} = \sqrt{\frac{1}{3}}$, which shows that the equilibrium probability of government downfall is non-monotonic in t.

The Mobilization Effect Game. In the mobilization effect game, given that the equilibrium actions are $p^*(t) = r^*(t) = \frac{t}{1+t}$[2], the equilibrium probability of government downfall is

$$[1 - S(r^*(t))]G(p^*(t))) = \frac{t}{(1+t)^2},$$

which implies that the effect of a change in t on the equilibrium probability of government downfall is given by the following expression:

$$\frac{\delta}{\delta t}\{[1 - S(r^*)]G(p^*))\} = \frac{1-t}{(1+t)^3}.$$

Because $t > 0$, the equilibrium probability of government downfall increases in t if $1 - t > 0$ and decreases in t otherwise. This implies that the equilibrium probability of government downfall increases in t if and only if

$$t \leq \bar{t}' = 1,$$

and decreases in t if $t > \bar{t}'$, which shows that the equilibrium probability of government downfall is non-monotonic in t.

[2]Again, notice that for any $t > 0$, the equilibrium actions are always interior, i.e., $0 < r^* < 1$ and $0 < p^* < 1$.

The Preventive Control Effect Game. In the preventive control effect game, the equilibrium probability of government downfall is given by the following expression:

$$[1 - S(r^*(t))]G(p^*(t)) = \frac{1}{(1+t)^2},$$

and a simple inspection of the above expression shows that it is always decreasing in t as claimed. $\square$

Sequential Game with Imperfect Observability of Action

In this section, we show that our main results can be obtained in a setting in which the government moves first while the opposition imperfectly observes the level of r before choosing its action (similarly, our results are robust in a setting in which the opposition moves first while the government imperfectly observes the level of p before choosing its action).

To do so, we analyze a variant of the game in which the government moves first and chooses r and then, before choosing p, the opposition observes a signal $\hat{r}$ that is correlated but not perfectly so with the true level of government preventive repression. To formalize this notion, let $\hat{r} = r + \eta$ and $\eta \sim N(0, \sigma_\eta^2)$, where $\sigma_\eta \neq 0$. Notice that this specification does not impose any restrictions on how precise is the opposition's signal (i.e., the noise parameter σ_η) about the (true) level of government preventive repression only that $\sigma_\eta \neq 0$; σ_η can be low, which means that the opposition knows the level of preventive repression with high accuracy or it can be high, which means that the opposition knows the level of preventive repressive activities with low accuracy.

In this game, a pure strategy of the opposition is a function $p(\hat{r})$ that maps $\hat{r}$ into actions. The next proposition shows that to analyze the equilibria of the game in which the opposition observes a signal $\hat{r}$ before choosing its action, it suffices to analyze the equilibria of the game in which the government and the opposition makes their choices simultaneously.[3] We have the following result.

[3]Bagwell (1995) has pointed out the equivalence between the equilibria of the game with imperfect observability of the first mover's action and the equilibria of the simultaneous interaction in a seminal analysis on the micro-foundations of commitment (Bagwell 1995).

Proposition 9. *For any $\sigma_\eta \neq 0$, a pair of strategies $\{r^*, p^*(\hat{r})\}$ is a pure strategy perfect Bayesian equilibrium of the game if and only if $p^*(\hat{r}) = p^*$ for all $\hat{r}$, and $\{r^*, p^*\}$ is a pure strategy equilibrium of the game in which the government and the opposition choose their action simultaneously.*

Proof. First, let us consider the if-part of the proposition. If the government plays a pure strategy r^* in equilibrium, the opposition's belief must place probability 1 on the event that the government chooses action r^*, regardless of the specific signal $\hat{r}$ that the community observes (any $\hat{r}$ is consistent with the government choosing r^* since $\sigma_\eta \neq 0$). Given this belief, action p^* maximizes the opposition's expected utility since p^* is an optimal response to r^* in the simultaneous game. Finally, the government's optimal action is r^* given that the opposition chooses p^* for any signal $\hat{r}$. This show that, for all $\hat{r}$, $\{r^*, p^*(\hat{r}) = p^*\}$ is a pure strategy perfect Bayesian equilibrium of the game in which the government moves first while the opposition imperfectly observes the level of r before choosing its action.

Next, let us first consider the only-if-part of the statement. Contrary to the claim, let us assume that $r = r'$ in a perfect Bayesian equilibrium of the sequential game, and that r' is not part of any pure strategy equilibrium of the simultaneous game. Because we have a pure strategy equilibrium, the opposition's belief must place probability 1 on r', and, as a result, $p(\hat{r}) = p(r')$ for all $\hat{r}$ (where $p(\cdot)$ is the optimal response function from the simultaneous game). Because the opposition's action $p(r')$ is independent of $\hat{r}$, it must be the case that r' maximizes the government's utility, given that the opposition plays $p(r')$, that is, $r' = r(p(r'))$. But this contradicts the assumption that $\{r', p(r')\}$ is not a pure strategy equilibrium of the simultaneous game.

□

The above proposition shows that the only pure strategy equilibria in the sequential game in which the opposition observes the level of preventive repression activities with some noise are the ones of the simultaneous game. The rationale is as follows: in any pure strategy equilibrium, the opposition correctly anticipates the government's equilibrium action, and therefore its own equilibrium action is independent of the observed $\hat{r}$. But, then, if the government anticipates that the opposition's equilibrium action is independent of $\hat{r}$ the opposition observes, the government chooses its best response for the given opposition's equilibrium action.

Thus, only actions r^* and p^* that are best replies to each other can arise on the equilibrium path of the game in which the opposition imperfectly observes the government's action before choosing its action. Since the simultaneous action game has a unique pure strategy equilibrium, proposition 9 implies that the sequential game with imperfect observability of action has the same pure strategy equilibrium. Therefore, all our results presented in the main text obtain in this sequential game with imperfect observability of action.

Also, an identical proof would show that the unique strategy equilibrium in the sequential game in which the opposition moves first while the government observes the opposition's action with some noise is the same as the unique pure strategy equilibrium of the simultaneous game.