

# Collective Action and Constraints on Repression at the Endgame

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## Appendix: Proofs of Propositions

In this Appendix, we provide the proofs for Propositions 3 and 4 (notice that Propositions 1 and 2 are proved in the text).

*Proof of Proposition 3.* Given  $\bar{e}$ , a security agent's marginal utility with respect to  $e_i$  is

$$\Pi(\bar{e})B'_A(e_i) - [1 - \Pi(\bar{e})]C'_A(e_i). \quad (1)$$

If  $\Pi(\bar{e})B'_A(e_i) - [1 - \Pi(\bar{e})]C'_A(e_i) < 0$  for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ , then the unique optimal level of repressive effort is  $e_i = e_l$  for all  $i$  and hence the unique equilibrium level of effort is  $e^* = e_l$ . Below we derive a sufficient condition on the exogenous parameters such that the expression (1) is negative for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ . Notice that the condition for expression (1) to be negative can re-written as follows:

$$\frac{\Pi(\bar{e})}{1 - \Pi(\bar{e})}B'_A(e_i) < C'_A(e_i).$$

The expression  $\frac{\Pi(\bar{e})}{1 - \Pi(\bar{e})}$  is increasing in  $\bar{e}$  and attains the maximum value at  $e_h$ . Also, the marginal benefit  $B'_A(e_i)$  is decreasing in  $e_i$  and attains the maximum value at  $e_l$ . That is,  $\frac{\Pi(\bar{e})}{1 - \Pi(\bar{e})}B'_A(e_i) \leq \frac{\Pi(e_h)}{1 - \Pi(e_h)}B'_A(e_l)$  for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ . Let  $\bar{C} \equiv \frac{\Pi(e_h)}{1 - \Pi(e_h)}B'_A(e_l)$ .<sup>1</sup> Finally, the marginal cost  $C'_A(e_i)$  is increasing in  $e_i$  and thus attains the minimum value at  $e_l$ . If  $C'_A(e_l) > \bar{C}$ , the expression (1) is negative for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ , and therefore  $e^* = e_l$  is the unique equilibrium level of effort in the repression subgame.

If  $\Pi(\bar{e})B'_A(e_i) - [1 - \Pi(\bar{e})]C'_A(e_i) > 0$  for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ , then the unique optimal level of repressive effort is  $e_i = e_h$  for all  $i$  and hence the unique equilibrium level of effort is  $e^* = e_h$ . Below we derive a sufficient condition on the exogenous parameters such that expression (1) is positive for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ . The condition for expression (1) to be positive can re-written as follows:

$$\frac{\Pi(\bar{e})}{1 - \Pi(\bar{e})}B'_A(e_i) > C'_A(e_i).$$

The expression  $\frac{\Pi(\bar{e})}{1 - \Pi(\bar{e})}$  is increasing in  $\bar{e}$  and attains the minimum value at  $e_l$ . Also, the marginal benefit  $B'_A(e_i)$  is decreasing in  $e_i$  and attains the minimum value at  $e_h$ . That is,  $\frac{\Pi(\bar{e})}{1 - \Pi(\bar{e})}B'_A(e_i) \geq \frac{\Pi(e_l)}{1 - \Pi(e_l)}B'_A(e_h)$  for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ . Let  $\underline{C} \equiv \frac{\Pi(e_l)}{1 - \Pi(e_l)}B'_A(e_h)$ .<sup>2</sup> Finally, the marginal cost  $C'_A(e_i)$  strictly increases in  $e_i$  and attains the maximum value at

<sup>1</sup>Notice that  $\bar{C}$  is well-defined since  $\Pi(e_h) < 1$

<sup>2</sup>Notice that  $\underline{C}$  is well-defined since  $\Pi(\cdot)$  is increasing in  $\bar{e}$  and  $\Pi(e_h) < 1$ .

$e_h$ . If  $C'_A(e_h) < \underline{C}$ , expression (1) is positive for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ , and therefore  $e^* = e_h$  is the unique equilibrium level of effort in the repression subgame.  $\square$

*Proof of Proposition 4.* Given  $\bar{e}$ , a security agent's marginal utility with respect to  $e_i$  is given by expression (1). If  $\Pi(\bar{e})B'_A(e_i) - [1 - \Pi(\bar{e})]C'_A(e_i) < 0$  for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ , then the unique optimal level of repressive effort is  $e_i = e_l$  and hence the unique equilibrium level of repressive effort is  $e^* = e_l$ . Below we derive a sufficient condition on the (marginal) benefit of repression such that expression (1) is negative for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ . The condition for expression (1) to be negative can re-written as follows:

$$B'_A(e_i) < \frac{1 - \Pi(\bar{e})}{\Pi(\bar{e})} C'_A(e_i).$$

The expression  $\frac{1 - \Pi(\bar{e})}{\Pi(\bar{e})}$  is decreasing in  $\bar{e}$  and attains the minimum value at  $e_h$ . Also, the marginal cost  $C'_A(e_i)$  is increasing in  $e_i$  and attains the minimum value at  $e_l$ . That is,  $\frac{1 - \Pi(\bar{e})}{\Pi(\bar{e})} C'_A(e_i) \geq \frac{1 - \Pi(e_h)}{\Pi(e_h)} C'_A(e_l)$  for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ . Let  $\underline{B} \equiv \frac{1 - \Pi(e_h)}{\Pi(e_h)} C'_A(e_l)$ .<sup>3</sup> Finally, the marginal benefit  $B'_A(e_i)$  strictly decreases in  $e_i$  and attains the maximum value at  $e_l$ . If  $B'_A(e_l) < \underline{B}$ , then expression (1) is negative for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ , and therefore  $e^* = e_l$  is the unique equilibrium level of effort in the repression subgame.

If  $\Pi(\bar{e})B'_A(e_i) - [1 - \Pi(\bar{e})]C'_A(e_i) > 0$  for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ , then the unique optimal level of repressive effort is  $e_i = e_h$  and hence the unique equilibrium level of effort is  $e^* = e_h$ . Below we derive a sufficient condition on the primitives as a function of the marginal benefit of repression such that expression (1) is positive for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ . The condition for expression (1) to be positive can re-written as follows:

$$B'_A(e_i) > \frac{1 - \Pi(\bar{e})}{\Pi(\bar{e})} C'_A(e_i).$$

The expression  $\frac{1 - \Pi(\bar{e})}{\Pi(\bar{e})}$  is decreasing in  $\bar{e}$  and attains the maximum value at  $e_l$ . Also, the marginal cost  $C'_A(e_i)$  is increasing in  $e_i$  and attains the maximum value at  $e_h$ . That is,  $\frac{1 - \Pi(\bar{e})}{\Pi(\bar{e})} C'_A(e_i) \leq \frac{1 - \Pi(e_l)}{\Pi(e_l)} C'_A(e_h)$  for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ . Let  $\overline{B} \equiv \frac{1 - \Pi(e_l)}{\Pi(e_l)} C'_A(e_h)$ .<sup>4</sup> Finally, the marginal benefit  $B'_A(e_i)$  strictly decreases in  $e_i$  and attains the minimum value at  $e_h$ . If  $B'_A(e_h) > \overline{B}$ , then expression (1) is positive for all  $e_i \in [e_l, e_h]$  and all  $\bar{e} \in [e_l, e_h]$ , and therefore  $e^* = e_h$  is the unique equilibrium level of effort in the repression subgame.  $\square$

<sup>3</sup>Notice that  $\underline{B}$  is well-defined since  $\Pi(e_l) > 0$  and  $\Pi(\cdot)$  is increasing in  $\bar{e}$ .

<sup>4</sup>Notice that  $\overline{B}$  is well-defined since  $\Pi(e_l) > 0$ .